# Dominance-Based Rough Set Approach to Multiple Criteria Ranking with Sorting-Specific Preference Information

**Miłosz Kadziński, Roman Słowiński and Marcin Szeląg**

**Abstract** A novel multiple criteria decision aiding method is proposed, that delivers a recommendation characteristic for ranking problems but employs preference information typical for sorting problems. The method belongs to the category of ordinal regression methods: it starts with preference information provided by the Decision Maker (DM) in terms of decision examples, and then builds a preference model that reproduces these exemplary decisions. The ordinal regression is analogous to inductive learning of a model that is true in the closed world of data where it comes from. The sorting examples show an assignment of some alternatives to pre-defined and ordered quality classes. Although this preference information is purely ordinal, the number of quality classes separating two assigned alternatives is meaningful for an ordinal intensity of preference. Using an adaptation of the Dominance-based Rough Set Approach (DRSA), the method builds from this information a decision rule preference model. This model is then applied on a considered set of alternatives to finally rank them from the best to the worst. The decision rule preference model resulting from DRSA is able to represent the preference information about the ordinal intensity of preference without converting this information into a cardinal scale. Moreover, the decision rules can be interpreted straightforwardly by the DM, facilitating her understanding of the feedback between the preference information and the preference model. An illustrative case study performed in this paper supports this claim.

**Keywords** Decision analysis · Preference learning · Ranking · Dominance-based rough set approach · Decision rules · Assignment examples

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# 1 Introduction

Decision problems considered in Multiple Criteria Decision Aiding (MCDA) concern a set of alternatives evaluated on a consistent family of criteria. MCDA gives the decision makers some tools and methods for structuring the problem, preference handling, and carrying forward the process of decision making. Taking into account the type of expected results, the term decision can be interpreted in different ways. Generally, in MCDA we distinguish three types of decision problems: choice, ranking, and sorting.

In choice problems, one aims at selecting a small subset of potentially best alternatives. In ranking problems, alternatives should be ordered from the best to the worst. Finally, the sorting problem is about assigning the alternatives to some predefined and ordered classes. When considering multiple conflicting criteria, arriving at a recommendation for each type of decision problems requires the use of some particular decision aiding method [19].

Each MCDA method is distinguished by the type of admitted preference information, ways of constructing and exploiting the preference model, and techniques used to work out a recommendation. Usually, these methods are designed for dealing with either ranking and choice or sorting (ordinal classification) problems.

In this paper, we introduce a novel MCDA method that delivers recommendation characteristic for ranking problems but makes use of preference information that is typical for sorting problems. This method employs an adaptation of the Dominance-based Rough Set Approach (DRSA) (see [8, 11, 12, 21, 22]). Given the preference information in terms of class assignments (sorting) of some reference alternatives, it builds a decision rule preference model. This model is then applied on a set of alternatives to be ranked, yielding a recommendation in terms of a weak order of these alternatives.

The method proposed belongs to the category of ordinal regression methods that start with preference information provided by the Decision Maker (DM) in terms of decision examples, and then build a mathematical model that replicates these exemplary decisions. For this ability, the model is called DM's preference model.

The motivation behind the proposed approach to multiple criteria ranking is twofold:

- the preference information provided by the DM in terms of sorting examples permits to express the intensity of preference between alternatives in a purely ordinal way, such that intensity of preference of $a$ over $b$ is comparable to that of $c$ over $d$ only if the interval of classes between the assignment of $a$ and $b$ includes or is included in the interval of $c$ and $d$; otherwise the intensities are non-comparable;
- the decision rule preference model resulting from DRSA is able to express the preference relation with the above meaning of the ordinal intensity without any transformation of the input preference information; moreover, the decision rules can be interpreted straightforwardly by the DM, facilitating her understanding of the feedback between the preference information and the preference model.

Let us comment on these motivations in more detail. First, let us observe that when the final aim is to construct a ranking of all considered alternatives, the preference information has often the form of pairwise comparisons of some or all considered alternatives. This is quite natural, because position of each alternative in the ranking depends on result of its comparison with other alternatives. These pairwise comparisons often admit a multi-graded preference relation, expressing an intensity of preference.

For example, in the Analytical Hierarchy Process [20], the DM is supposed to compare pairwise all considered alternatives and express the intensity of preference on a pre-defined cardinal ratio scale. In the MACBETH method [1], all pairs of alternatives are assigned to some ordered classes of preference intensity and, finally, a cardinal intensity scale concordant with these assignments is computed. Some other methods do not require more from the DM than an ordinal expression of preference intensity, like "*a* is preferred to *b* at least as strongly as *c* is preferred to *d*", and obtain in consequence a single-graded quaternary relation in the set of pairs of alternatives; this is the case of the GRIP method [7], which builds a set of general additive value functions that replicate the ordinal preference information, and provides at the output necessary and possible quaternary relations that contribute to construction of necessary and possible rankings, respectively. All above methods use a value function preference model. Observe that rankings established by a value function permit to speak about intensity of preference between alternatives in the ranking, as the scale of the value function is an interval scale.

MCDA methods that use outranking relation preference model do not consider intensity of preference either in the input preference information, or in the resulting ranking, which has an ordinal character.

Methods based on logical representation of preferences in terms of monotonic decision rules, like DRSA, are able to process preference information with specified intensity of preference [8]. In this case, the pairwise comparisons of some reference alternatives get a degree of intensity of preference assigned by the DM. These degrees are linearly ordered, so that DRSA can approximate upward and downward unions of the degrees. Decision rules induced from these approximations suggest assignment of a pair of alternatives to a preference relation having at least or at most a specified degree of intensity. Application of these rules on a set of alternatives leads to a fuzzy preference graph, whose exploitation with a weighted fuzzy net flow score procedure leads to a final ranking. A difference of positions in this ranking does not have the meaning of intensity of preference, which is similar to rankings obtained by methods using as preference model an outranking relation.

Experience indicates, however, that answering the questions about the intensity of preference between two alternatives in cardinal terms requires too big cognitive effort on the part of the DM. Facilitating the DM's elicitation of the intensity of preference for pairwise comparisons is thus the first motivation of this paper. The method proposed in this paper permits the DM to express the intensity of preference between reference alternatives in a purely ordinal way, as assignments of alternatives to pre-defined and ordered quality classes. The order of these classes has no cardinal meaning, so that the number of classes separating two assigned alternatives is not meaningful for intensity of preference. The only comparison of intensities of preference is possible when an interval of classes for alternatives $a, b$ includes or is included in the interval of classes for alternatives $c, d$; precisely, the following eight situations of comparability for intensities of preference may occur:

- $(a, b) \succeq (c, d)$ if $a \succeq b$, $c \succeq d$, and the interval of classes for $a, b$ includes the interval of classes for $c, d$;
- $(a, b) \succeq (c, d)$ if $a \succeq b$, $c \preceq d$, and the interval of classes for $a, b$ includes the interval of classes for $c, d$;
- $(a, b) \succeq (c, d)$ if $a \succeq b$, $c \preceq d$, and the interval of classes for $a, b$ is included in the interval of classes for $c, d$;
- $(a, b) \succeq (c, d)$ if $a \preceq b$, $c \preceq d$, and the interval of classes for $a, b$ is included in the interval of classes for $c, d$;
- $(a, b) \preceq (c, d)$ if $a \preceq b$, $c \succeq d$, and the interval of classes for $a, b$ includes the interval of classes for $c, d$;
- $(a, b) \preceq (c, d)$ if $a \preceq b$, $c \preceq d$, and the interval of classes for $a, b$ includes the interval of classes for $c, d$;
- $(a, b) \preceq (c, d)$ if $a \succeq b$, $c \succeq d$, and the interval of classes for $a, b$ is included in the interval of classes for $c, d$;
- $(a, b) \preceq (c, d)$ if $a \preceq b$, $c \succeq d$, and the interval of classes for $a, b$ is included in the interval of classes for $c, d$.

The second motivation is related to the choice of the preference model type. We chose the logical preference model in terms of monotonic decision rules. This is because axiomatic analysis of all three preference model types leads to the conclusion that decision rules, as they are defined in DRSA, are the only aggregation operators that give account of most complex interactions among criteria, are non-compensatory, accept ordinal evaluation scales and do not convert ordinal evaluations into cardinal ones [14]. Rules identify values that drive DM's decisions—each rule is a scenario of a causal relationship between evaluations on a subset of criteria and a comprehensive judgment. They are also easily interpretable by users who trust proposed recommendations more [13].

In this introduction, we should also refer to ranking methods based on preference learning in a way proposed by Machine Learning (ML) [10]. In ML, this task is known as "learning to rank" and also involves learning of a preference model from pairwise comparisons of some alternatives (called items in ML) [5, 9, 18]. Precisely, the pairwise comparisons are provided by users (DMs) as lists of items with some partial order between items in each list. This information is called the training data. Machine preference learning consists in discovering a model that predicts preference for a new set of items (or the input set of items considered in a different context) so that the produced ranking is statistically "similar" to the order provided as the training data. In this approach, learning is traditionally achieved by minimizing an empirical estimate of an assumed loss function on rankings [6]. Learning to rank emerged to address application needs in areas such as information retrieval, Internet-related applications, and bio-informatics. Indeed, ranking is at the core of document retrieval, collaborative filtering, or computational advertising. In recommender systems, a ranked list of related items should be recommended to a user who has shown interest in some other items. In computational biology, one ranks candidate structures in protein structure prediction problem, whereas in proteomics there is a need for the identification of frequent top scoring peptides.

MCDA and machine preference learning show many similarities, however, there are also striking differences between them [4]. In particular, MCDA stimulates the DM to interact with the method by incrementally enriching the preference information and observing its consequences on the recommended rankings. This feature reveals a specific aspect of learning adopted in MCDA which contrasts with ML oriented towards preference discovery without interaction with the DM. For the purpose of this article, we should also stress that neither in MCDA nor in machine preference learning, the ordinal intensity of preference has been considered in the way explained above as the first motivation.

The paper is organized as follows. In Sect. 2, we define notation and basic concepts related to the preference information and approximated sets of pairs of alternatives. Induction of decision rules from rough approximations is described in Sect. 3. Application of decision rules on a set of alternatives is a subject of Sect. 4. In Sect. 5, exploitation of a preference graph resulting from application of decision rules is presented together with the end result in form of a weak order on the set of alternatives. In Sect. 6, an illustrative example shows how the proposed method can be applied in a hypothetical case study. The final section includes summary and conclusions.

## 2 Notation and Basic Concepts

We shall use the following notation:

- $A = \{x, y, \ldots\}$—a finite set of alternatives to be ranked;
- $C_1, C_2, \ldots, C_p$—$p$ predefined preference-ordered classes, where $C_{h+1}$ is preferred to $C_h$, $h = 1, \ldots, p-1$; moreover, $H = \{1, \ldots, p\}$ denotes the set of class indices;
- $A^R = \{a, b, \ldots\}$—a finite set of $m$ reference alternatives, on which the DM accepts to express holistic preferences, such that each reference alternative is assigned to one of the classes $C_1, C_2, \ldots, C_p$; we assume that $A^R \subseteq A$;
- $B = A^R \times A^R$;
- $G = \{g_1, g_2, \ldots, g_j, \ldots, g_n\}$—a finite set of $n$ evaluation criteria with ordinal or cardinal scales; without loss of generality, we assume that all criteria are of gain type, i.e., the greater the criterion value, the better.

A criterion with the cardinal scale is called a cardinal criterion; the set of all cardinal criteria is denoted by $G^N \subseteq G$, while $\mathcal{J}_{G^N}$ denotes their indices. A criterion with the ordinal scale is called an ordinal criterion; the set of all ordinal criteria is denoted by $G^O \subseteq G$, while $\mathcal{J}_{G^O}$ denotes their indices. Moreover, $G^N \cup G^O = G$ and $G_N \cap G_O = \emptyset$. For the sake of simplicity, we assume that for each cardinal criterion $g_j \in G^N$, intensity of preference of $a$ over $b$ is defined as the difference of evaluations, i.e., it is equal to $\Delta_j(a, b) = g_j(a) - g_j(b)$. In case of criterion $g_j \in G^O$ with an ordinal scale, one can only establish an order of evaluations $g_j(a)$, $a \in A$.

**Preference information.** We assume that the DM provides a set of assignment examples, each one consisting of a reference alternative $a \in A^R$ and its assignment $Cl_{DM}(a) = C_i$, $1 \leq i \leq p$.

**Dominance relation for pairs of alternatives.** The pair of alternatives $(a, b) \in B$ *dominates* pair $(c, d) \in B$ with respect to set of criteria $G$, denoted by $(a, b)D_2(c, d)$, if and only if (iff):

- for all $g_j \in G^N$, $\Delta_j(a, b) \geq \Delta_j(c, d)$, where $\Delta_j(a, b) = g_j(a) - g_j(b)$;
- for all $g_j \in G^O$, $g_j(a) \geq g_j(c)$ and $g_j(b) \leq g_j(d)$.

Dominance relation $D_2$ is a partial weak order on $B$. If $(a, b)D_2(c, d)$, one expects that not $Cl_{DM}(c) \geq Cl_{DM}(a)$ and $Cl_{DM}(d) \leq Cl_{DM}(b)$, with at least one of these relations being strict. Violation of this principle is considered as an inconsistency with respect to dominance relation $D_2$ on $B$ and the order imposed on considered classes.

**Granules of knowledge.** The set of pairs of alternatives dominating $(a, b) \in B$, $D_2^+(a, b)$, is called the *dominating set* or *positive dominance cone*:

$$D_2^+(a, b) = \{(c, d) \in B : (c, d)D_2(a, b)\}. \tag{1}$$

The set of pairs of alternatives dominated by $(a, b)$, $D_2^-(a, b)$, is called the *dominated set* or *negative dominance cone*:

$$D_2^-(a, b) = \{(c, d) \in B : (a, b)D_2(c, d)\}. \tag{2}$$

**Approximated sets of pairs of alternatives.** Since classes $C_1, \ldots, C_p$ are preference-ordered, when comparing two reference alternatives $a, b \in A^R$, where $a \in C_i$ and $b \in C_j$, $1 \leq i, j \leq p$, three possibilities may arise:

- $i = j$, which means that $a$ is indiscernible with $b$,
- $i > j$, which means that $a$ is preferred to $b$,
- $i < j$, which means that $b$ is preferred to $a$.

Notice that the comparison of two reference alternatives has an ordinal character only. In consequence, given two pairs of reference alternatives $(a, b), (c, d) \in B$, one can compare them with respect to preference only if the interval of classes to which $a, b$ belong includes (or is included in) the interval of classes to which $c, d$ belong. Specifically, if $Cl_{DM}(a) \geq Cl_{DM}(c)$ and $Cl_{DM}(b) \leq Cl_{DM}(d)$, then $a$ is preferred to $b$ at least as much as $c$ is preferred to $d$ ($c$ is preferred to $d$ at most as much as $a$ is preferred to $b$). Otherwise, these pairs are incomparable.

Let us consider the following set of pairs of alternatives

$$S^{i,j} = \{(a, b) \in B : Cl_{DM}(a) = C_i \text{ and } Cl_{DM}(b) = C_j\}, \tag{3}$$

where $1 \leq i, j \leq p$. It includes pairs of alternatives with the same preference of the first alternative over the second one.

Using definition (3), one can define the following unions of sets:

$$S^{\succeq(i,j)} = \bigcup_{k \geq i, l \leq j} S^{k,l}; \tag{4}$$

$$S^{\preceq(i,j)} = \bigcup_{k \leq i, l \geq j} S^{k,l}, \tag{5}$$

where $1 \leq i, j, k, l \leq p$. Union $S^{\succeq(i,j)}$ is a set composed of pairs of reference alternatives $(a, b)$ such that $a$ is preferred to $b$ at least as much as $c$ is preferred to $d$, where $(c, d) \in S^{i,j}$. Analogously, union $S^{\preceq(i,j)}$ is a set composed of pairs of reference alternatives $(a, b)$ such that $a$ is preferred to $b$ at most as much as $c$ is preferred to $d$, where $(c, d) \in S^{i,j}$.

Notice that the above unions are binary relations, thus, the expressions $(a, b) \in S^{\succeq(i,j)}$ and $aS^{\succeq(i,j)}b$ can be used alternatively. Moreover, the unions are related in the following way: $S^{\succeq(i,j)} \supseteq S^{\succeq(k,l)}$ if and only if $i \leq k$ and $j \geq l$, for all $i, j, k, l \in H$; analogously, $S^{\preceq(i,j)} \supseteq S^{\preceq(k,l)}$ if and only if $i \geq k$ and $j \leq l$, for all $i, j, k, l \in H$. Thus, there exist two lattices of unions $S^{\succeq(i,j)}$ and $S^{\preceq(i,j)}$, respectively, ordered by weak inclusion relation. These lattices can be depicted by Hasse diagrams.

Figure 1 presents Hasse diagram of lattice of unions $S^{\succeq(i,j)}$, while Fig. 2 presents Hasse diagram of lattice of unions $S^{\preceq(i,j)}$, $1 \leq i, j \leq 4$; in both diagrams arcs show the direction of inclusion, i.e., an arc leading from union $U_1$ to $U_2$ marks inclusion $U_1 \subseteq U_2$. For example, in case of union $S^{\succeq(3,2)}$:

$$S^{\succeq(3,2)} \supseteq S^{\succeq(3,1)} \supseteq S^{\succeq(4,1)} \text{ and}$$
$$S^{\succeq(3,2)} \supseteq S^{\succeq(4,2)} \supseteq S^{\succeq(4,1)}.$$

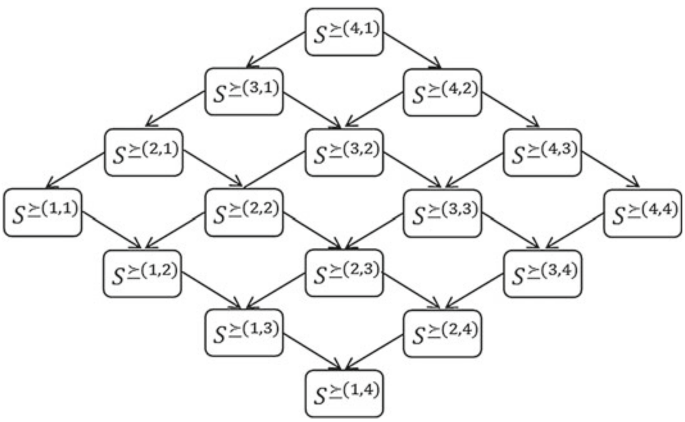


**Fig. 1** Hasse diagram of the lattice of unions $S^{\succeq(i,j)}$ for $1 \leq i, j \leq 4$

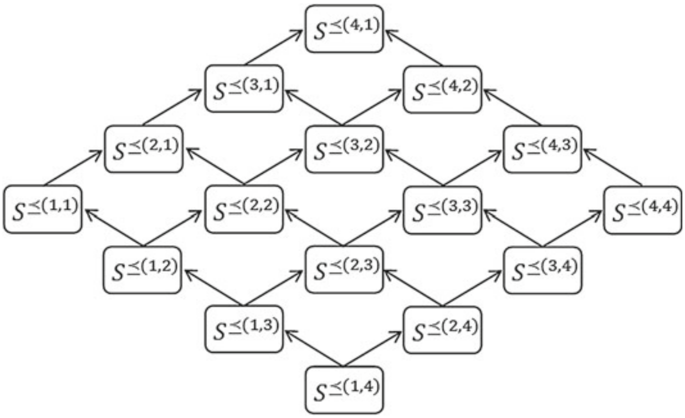


**Fig. 2** Hasse diagram of the lattice of unions $S^{\preceq(i,j)}$ for $1 \leq i, j \leq 4$

Moreover, in case of union $S^{\preceq(2,2)}$:

$$S^{\preceq(2,2)} \supseteq S^{\preceq(1,2)} \supseteq S^{\preceq(1,3)} \supseteq S^{\preceq(1,4)} \text{ and}$$
$$S^{\preceq(2,2)} \supseteq S^{\preceq(2,3)} \supseteq S^{\preceq(1,3)} \supseteq S^{\preceq(1,4)} \text{ and}$$
$$S^{\preceq(2,2)} \supseteq S^{\preceq(2,3)} \supseteq S^{\preceq(2,4)} \supseteq S^{\preceq(1,4)}.$$

**Rough approximations.** We approximate unions $S^{\succeq(i,j)}$ using positive dominance cones $D_2^+(\cdot,\cdot)$, and unions $S^{\preceq(i,j)}$ using negative dominance cones $D_2^-(\cdot,\cdot)$. The lower and upper approximations of $S^{\succeq(i,j)}$ and $S^{\preceq(i,j)}$ are defined, respectively, as:

$$\underline{S^{\succeq(i,j)}} = \{(a,b) \in S^{\succeq(i,j)} : D_2^+(a,b) \cap (S^{\preceq(i-1,j)} \cup S^{\preceq(i,j+1)}) = \emptyset\}; \quad (6)$$
$$\overline{S^{\succeq(i,j)}} = \{(a,b) \in B : D_2^-(a,b) \cap S^{\succeq(i,j)} \neq \emptyset\}; \quad (7)$$
$$\underline{S^{\preceq(i,j)}} = \{(a,b) \in S^{\preceq(i,j)} : D_2^-(a,b) \cap (S^{\succeq(i,j-1)} \cup S^{\succeq(i+1,j)}) = \emptyset\}; \quad (8)$$
$$\overline{S^{\preceq(i,j)}} = \{(a,b) \in B : D_2^+(a,b) \cap S^{\preceq(i,j)} \neq \emptyset\}, \quad (9)$$

where $1 \leq i, j \leq p$.

Finally, the boundaries of $S^{\succeq(i,j)}$ and $S^{\preceq(i,j)}$ are defined, respectively, as:

$$Bn(S^{\succeq(i,j)}) = \overline{S^{\succeq(i,j)}} \setminus \underline{S^{\succeq(i,j)}}, \quad (10)$$
$$Bn(S^{\preceq(i,j)}) = \overline{S^{\preceq(i,j)}} \setminus \underline{S^{\preceq(i,j)}}. \quad (11)$$

Let us explain the idea underlying definitions of $\underline{S^{\succeq(i,j)}}$ and $\overline{S^{\succeq(i,j)}}$. On one hand, $\underline{S^{\succeq(i,j)}}$ contains pairs of reference alternatives $(a,b) \in B$ which are not dominated by any pair of reference alternatives $(c,d) \in B$ such that the class of $c$ is worse (worse or equal) than that of $a$ and the class of $d$ is better or equal (better) than that of $b$. For example, the lower approximation of $S^{\succeq(3,2)}$ contains pairs of reference alternatives $(a, b)$ which are not dominated by any pair $(c, d)$ belonging to $S^{\preceq(2,2)}$ or $S^{\preceq(3,3)}$. On the other hand, $\overline{S^{\succeq(i,j)}}$ contains pairs of reference alternatives $(a, b) \in B$ which dominate at least one pair of reference alternatives $(c, d)$ belonging to $S^{\succeq(i,j)}$.

## 3 Induction of Decision Rules

We assume that a preference model of the DM is a set of minimal decision rules, being statements of the type: "*if premise, then conclusion*" that represent a form of dependency between the condition part and the decision part. The premise of a rule is a conjunction of elementary conditions concerning individual criteria, and the decision part of a rule suggests that a pair of alternatives covered by the rule should be assigned to particular union $S^{\succeq(i,j)}$ or $S^{\preceq(i,j)}$, $1 \leq i, j \leq p$. We say that a pair of alternatives is *covered* by a decision rule if it matches the premise of the rule. On the other hand, a pair of alternatives *supports* a decision rule if it matches both premise and conclusion of the rule. Although we can distinguish certain, possible, and approximate rules, in this paper we will focus on the certain rules only.

In order to induce certain decision rules with conclusion $xS^{\succeq(i,j)}y$ or $xS^{\preceq(i,j)}y$, $1 \leq i, j \leq p$, one needs to consider:

- *positive examples*, i.e., consistent pairs of reference alternatives concordant with given conclusion (pairs of reference alternatives from the lower approximation $\underline{S^{\succeq(i,j)}}$ or $\underline{S^{\preceq(i,j)}}$, respectively), and
- *negative examples*, i.e., pairs of reference alternatives contained in $S^{\preceq(i-1,j)} \cup S^{\preceq(i,j+1)}$ or $S^{\succeq(i,j-1)} \cup S^{\succeq(i+1,j)}$, respectively.

Observe that sets of positive and negative examples do not make partition of $B$. Apart from both types of examples, $B$ includes also so-called *neutral examples*, i.e., pairs of reference alternatives that belong to $B \setminus (S^{\succeq(i,j)} \cup S^{\preceq(i-1,j)} \cup S^{\preceq(i,j+1)})$ or $B \setminus (S^{\preceq(i,j)} \cup S^{\succeq(i,j-1)} \cup S^{\succeq(i+1,j)})$, respectively. These examples are not taken into account during rule induction.

In the following, when defining the syntax of decision rules, instead of concise conclusion $xS^{\succeq(i,j)}y$, we employ the equivalent expression $Cl_{DM}(x) \geq C_i$ and $Cl_{DM}(y) \leq C_j$, which is more informative for the DM. For the same reason, instead of conclusion $xS^{\preceq(i,j)}y$, we use the expression $Cl_{DM}(x) \leq C_i$ and $Cl_{DM}(y) \geq C_j$.

We distinguish two types of decision rules:

- "at least" decision rules, with the following syntax:

$$\begin{aligned}\text{if } \Delta_{j_1}(x, y) \geq \delta_{j_1} \text{ and } \ldots \text{ and } \Delta_{j_v}(x, y) \geq \delta_{j_v} \text{ and } \ldots \text{ and } g_{j_{v+1}}(x) \geq r_{j_{v+1}} \\ \text{and } g_{j_{v+1}}(y) \leq s_{j_{v+1}} \text{ and } \ldots \text{ and } g_{j_z}(x) \geq r_{j_z} \text{ and } g_{j_z}(y) \leq s_{j_z} \\ \text{then } Cl_{DM}(x) \geq C_i \text{ and } Cl_{DM}(y) \leq C_j,\end{aligned}$$

- "at most" decision rules, with the following syntax:

$$\text{if } \Delta_{j_1}(x,y) \le \delta_{j_1} \text{ and } \ldots \text{ and } \Delta_{j_v}(x,y) \le \delta_{j_v} \text{ and } \ldots \text{ and } g_{j_{v+1}}(x) \le r_{j_{v+1}} \\ \text{and } g_{j_{v+1}}(y) \ge s_{j_{v+1}} \text{ and } \ldots \text{ and } g_{j_z}(x) \le r_{j_z} \text{ and } g_{j_z}(y) \ge s_{j_z} \\ \text{then } Cl_{DM}(x) \le C_i \text{ and } Cl_{DM}(y) \ge C_j,$$

where $\delta_{j_k} \in \{g_{j_k}(a) - g_{j_k}(b) : (a,b) \in B\} \subseteq \Re$, for $j_k \in \{j_1, \ldots, j_v\} \subseteq \mathcal{J}_{G^N}$; $(r_{j_k}, s_{j_k}) \in \{(g_{j_k}(a), g_{j_k}(b)) : (a,b) \in B\} \subseteq \Re \times \Re$, for $j_k \in \{j_{v+1}, \ldots, j_z\} \subseteq \mathcal{J}_{G^O}$. For instance, considering ranking of cars, a decision rule could be "if car $x$ has maximum speed at least 25 km/h greater than car $y$ (cardinal criterion), and car $x$ has comfort at least 3 while car $y$ has comfort at most 2 (ordinal criterion), then car $x$ is assigned to class at least $C_3$ while car $y$ is assigned to class at most $C_1$", where values 2 and 3 code ordinal evaluations 'medium' and 'good', respectively.

The sets of "at least" and "at most" decision rules with the above syntax can be induced using one of the well-known rule induction algorithms, e.g., VC-DomLEM algorithm [2, 3], DomLEM algorithm [15, 23] or LEM2 algorithm [16, 17].

## 4 Application of Decision Rules

After induction of decision rules, the next step of the proposed methodology for multiple criteria ranking is the application of induced rules on set $A$. This application yields a preference structure on set $A$. Each pair of alternatives $(x, y) \in A \times A$ can be covered by some decision rules suggesting assignment to relations $S^{\succeq(i,j)}$ and/or to relations $S^{\preceq(i,j)}$, $1 \le i, j \le p$. It can be also not covered by any rule. In order to represent these situations, we introduce the following binary relations on $A$:

$$\mathbb{S}^{\succeq(i,j)} = \{(x,y) \in A \times A : \exists r \in R_{S^{\succeq(i,j)}} \text{ such that } r \text{ covers } (x,y)\}, \tag{12}$$

$$\mathbb{S}^{\preceq(i,j)} = \{(x,y) \in A \times A : \exists r \in R_{S^{\preceq(i,j)}} \text{ such that } r \text{ covers } (x,y)\}, \tag{13}$$

where $1 \le i, j \le p$ and $R_{S^{\succeq(i,j)}}$ denotes set of rules with conclusion $Cl_{DM}(x) \ge C_i$ and $Cl_{DM}(y) \le C_j$. Notice that $S^{\succeq(i,j)}$ and $\mathbb{S}^{\succeq(i,j)}$ are two different relations. The first one (see Definition (4)) is defined on set $A^R$ and concerns class assignments of reference alternatives, while the second one, introduced above, is defined on set $A$ and concerns coverage by induced decision rules.

The preference structure on $A$, composed of relations $\mathbb{S}^{\succeq(i,j)}$ and $\mathbb{S}^{\preceq(i,j)}$, $1 \le i, j \le p$, can be represented by a *preference graph*. It is a directed multigraph $\mathcal{G}$. Each vertex (node) $v_x$ of the preference graph corresponds to exactly one alternative $x \in A$. One can distinguish in $\mathcal{G}$ two types of arcs: $\mathbb{S}^{\succeq(i,j)}$-arcs and $\mathbb{S}^{\preceq(i,j)}$-arcs. For example, given $p = 4$, the preference graph features an $\mathbb{S}^{\succeq(4,1)}$-arc from vertex $v_x$ to $v_y$ iff $x\mathbb{S}^{\succeq(4,1)}y$. $\mathcal{G}$ is a multigraph since for any pair of alternatives $(x, y) \in A \times A$ there may be more than one arc from vertex $v_x$ to vertex $v_y$. A *final recommendation* for the multiple criteria ranking problem at hand, in terms of a complete weak order of all alternatives belonging to set $A$, can be obtained upon a suitable *exploitation* of the preference graph.

## 5 Exploitation of the Preference Graph

For the purpose of exploitation of the preference graph we employ the Weighted Fuzzy Net Flow Score (WFNFS) procedure described in [8]. As proved in [8], this procedure ensures that the obtained ranking contains dominance relation on set $A$, i.e., if alternative $a$ dominates $b$ with respect to set of criteria $G$, then $a$ is going to be ranked not lower than $b$. Since relations given by (12) and (13) are crisp, in the following we describe a simplified version of this procedure that we call Weighted Net Flow Score (WNFS) procedure.

Let [] denote the Iverson bracket function defined as:

$$[P] = \begin{cases} 1 \text{ if } P \text{ is true,} \\ 0 \text{ otherwise.} \end{cases} \tag{14}$$

In order to exploit a preference graph resulting from application of decision rules on set $A$, we employ *scoring function* $NFS : A \rightarrow \Re$ defined as

$$\begin{aligned} NFS(x) = \sum_{y \in A \setminus \{x\}} \Big( & \sum_{1 \le j \le i \le p} w^{\succeq(i,j)} \big([x \mathbb{S}^{\succeq(i,j)} y] - [y \mathbb{S}^{\succeq(i,j)} x]\big) \\ & - \sum_{1 \le i \le j \le p} w^{\preceq(i,j)} \big([x \mathbb{S}^{\preceq(i,j)} y] - [y \mathbb{S}^{\preceq(i,j)} x]\big)\Big), \end{aligned} \tag{15}$$

where weights $w^{\succeq(i,j)}$, for $i \geq j$, and weights $w^{\preceq(i,j)}$, for $i \leq j$, are by default equal to one but can be set different by the DM, e.g., in order to express greater importance of preference between alternatives from more distant classes. For each alternative $x \in A$, $NFS(x)$ takes into account two types of arguments in favor of $x$ (i.e., existence of $y \in A \setminus \{x\}$ such that $x \mathbb{S}^{\succeq(i,j)} y$, and existence of $y \in A \setminus \{x\}$ such that $y \mathbb{S}^{\preceq(i,j)} x$) and two types of arguments in disfavor of $x$ (i.e., existence of $y \in A \setminus \{x\}$ such that $y \mathbb{S}^{\succeq(i,j)} x$, and existence of $y \in A \setminus \{x\}$ such that $x \mathbb{S}^{\preceq(i,j)} y$). In the following, we will also use the notions of *strength* and *weakness* of an alternative, with respect to $\mathbb{S}^{\succeq(i,j)}$ and $\mathbb{S}^{\preceq(i,j)}$. For instance, the strength of $x \in A$ with respect to $\mathbb{S}^{\succeq(i,j)}$ is the value $\sum_{y \in A \setminus \{x\}} \sum_{1 \le j \le i \le p} w^{\succeq(i,j)} [x \mathbb{S}^{\succeq(i,j)} y]$, while the weakness of $x \in A$ with respect to $\mathbb{S}^{\preceq(i,j)}$ is the value $\sum_{y \in A \setminus \{x\}} \sum_{1 \le i \le j \le p} w^{\preceq(i,j)} [x \mathbb{S}^{\preceq(i,j)} y]$. Notice that if we put weights $w^{\succeq(i,j)}$ and $w^{\preceq(i,j)}$ on respective arcs of the considered preference graph, then calculation of $NFS(x)$ is equivalent to the calculation of a kind of net flow of vertex $v_x$ of this graph, where positive and negative inflows and outflows are considered. Function *NFS* induces a weak order on $A$, which is a solution of the considered multiple criteria ranking problem.

## 6 Illustrative Example

In this section, we illustrate the use of the presented method by considering real-world data concerning 13 Polish research units (group of joint evaluation, called SI3MU). The units are evaluated on the following three gain-type criteria:

- scientific activity ($g_1$) including scientific publications in international journals and number of patents; the evaluation reflects an average number of points gained by a single researcher of the unit;
- scientific potential ($g_2$) including the ability to grant scientific degrees, number of professor titles obtained by researchers of this unit in the evaluation period, as well as prestigious memberships of the researchers; all achievements are scored and these scores are summed up to get an evaluation;
- material effects of unit's activities ($g_3$) representing money acquired from grants or cooperation with industry.

The performances of the 13 considered research units (denoted by $a$ to $m$) are given in Table 1. The objective of the study is to order the units from the best to the worst. Although the aim consists in delivering a ranking, we will employ preference information which is specific for multiple criteria sorting problems.

**Assignment examples.** The preference information consists of exemplary class assignments for 6 randomly chosen reference units (these are distinguished with a non-empty entry in Table 1 (column $Ref.$)). There are 2 units assigned to each of the three considered classes $Cl_1$–$Cl_3$ (with $Cl_3$ being the best class). The assignment examples are derived from the original classification provided by the Polish Ministry of Science and Higher Education in 2014. Our aim is to "learn" a rule preference model on the 6 assignment examples, and apply this model on the whole set of 13 units in order to rank them.

**Rough approximations.** The provided 6 assignment examples entail consideration of 36 pairs of reference alternatives. It turns out that there is no inconsistency with respect to dominance relation $D_2$ on the set of these pairs and the order imposed on considered classes. The lower approximations for selected unions $S^{\succeq(i,j)}$ and $S^{\preceq(i,j)}$, $1 \leq i, j \leq 3$, are presented in Table 2. Obviously, in this case they are equal to upper approximations.

**Minimal sets of decision rules.** To induce decision rules from the lower approximations given in Table 2, we used a heuristic algorithm of a sequential covering type (inspired by LEM2). When selecting conditions for inclusion in a decision rule, we preferred these conditions that allowed to cover maximal number of positive examples, and then, to break possible ties, conditions which allowed to cover minimal number of negative examples. The resulting minimal sets of decision rules for selected unions are listed in Table 3. Each of them consists of just a single decision rule with at most two conditions.

**Table 1** Research units' performances and class assignments for reference units (column *Ref.*)

| Unit | $g_1$ | $g_2$ | $g_3$ | $Ref.$ |
|---|---|---|---|---|
| $a$ | 29.41 | 127 | 270.93 | $Cl_3$ |
| $b$ | 30.57 | 122 | 280.14 | – |
| $c$ | 31.34 | 283 | 122.78 | $Cl_3$ |
| $d$ | 46.46 | 117 | 34.44 | $Cl_2$ |
| $e$ | 15.99 | 50 | 155.55 | – |
| $f$ | 20.08 | 108 | 47.43 | $Cl_2$ |
| $g$ | 17.03 | 60 | 61.12 | – |
| $h$ | 23.65 | 150 | 27.22 | – |
| $i$ | 9.54 | 109 | 50.65 | – |
| $j$ | 11.41 | 106 | 28.39 | $Cl_1$ |
| $k$ | 10.98 | 2 | 13.28 | $Cl_1$ |
| $l$ | 9.66 | 0 | 11.69 | – |
| $m$ | 4.16 | 2 | 1.35 | – |

**Table 2** Lower approximations for selected unions $S^{\succeq(i,j)}$, $S^{\preceq(i,j)}$, $1 \leq i, j \leq 3$

| | | |
|---|---|---|
| $\underline{S}^{\succeq(3,1)}$ | = | $\{(a,j),(a,k),(c,j),(c,k)\}$ |
| $\underline{S}^{\succeq(3,2)}$ | = | $\{(a,j),(a,k),(c,j),(c,k),(a,d),(a,f),(c,d),(c,f)\}$ |
| $\underline{S}^{\succeq(2,1)}$ | = | $\{(a,j),(a,k),(c,j),(c,k),(d,j),(d,k),(f,j),(f,k)\}$ |
| $\underline{S}^{\succeq(3,3)}$ | = | $\{(a,j),(a,k),(c,j),(c,k),(a,d),(a,f),(c,d),(c,f),(a,a),(a,c),(c,a),(c,c)\}$ |
| $\underline{S}^{\succeq(2,2)}$ | = | $\{(a,j),(a,k),(c,j),(c,k),(a,d),(a,f),(c,d),(c,f),(d,j),(d,k),(f,j),(f,k),$ $(d,d),(d,f),(f,d),(f,f)\}$ |
| $\underline{S}^{\succeq(1,1)}$ | = | $\{(a,j),(a,k),(c,j),(c,k),(d,j),(d,k),(f,j),(f,k),(j,j),(j,k),(k,j),(k,k)\}$ |
| $\underline{S^{\preceq(1,3}})$ | = | $\{(j,a),(k,a),(j,c),(k,c)\}$ |
| $\underline{S}^{\preceq(2,3)}$ | = | $\{(j,a),(k,a),(j,c),(k,c),(d,a),(f,a),(d,c),(f,c)\}$ |
| $\underline{S}^{\preceq(1,2)}$ | = | $\{(j,a),(k,a),(j,c),(k,c),(j,d),(k,d),(j,f),(k,f)\}$ |
| $\underline{S}^{\preceq(3,3)}$ | = | $\{(j,a),(k,a),(j,c),(k,c),(d,a),(f,a),(d,c),(f,c)(a,a),(a,c),(c,a),(c,c)\}$ |
| $\underline{S}^{\preceq(2,2)}$ | = | $\{(j,a),(k,a),(j,c),(k,c),(d,a),(f,a),(d,c),(f,c),(j,d),(k,d),(j,f),(k,f),$ $(d,d),(d,f),(f,d),(f,f)\}$ |
| $\underline{S}^{\preceq(1,1)}$ | = | $\{(j,a),(k,a),(j,c),(k,c),(j,d),(k,d),(j,f),(k,f),(j,j),(j,k),(k,j),(k,k)\}$ |

**Weights of arcs in the preference graph.** To compute the score $NFS(x)$ of each unit $x \in A$, we employed the weights $w^{\succeq(i,j)}$ and $w^{\preceq(i,j)}$ given in Table 4. These weights were chosen to be symmetric, i.e., $w^{\succeq(i,j)} = w^{\preceq(j,i)}$. Moreover, they were set so that to express:

- greater importance of preference between units from more distant classes, e.g., $w^{\succeq(3,1)} = 6 > w^{\succeq(3,2)} = 3 > w^{\succeq(3,3)} = 1$;

**Table 3** Minimal sets of decision rules induced from lower approximations of selected unions $S^{\succeq(i,j)}$, $S^{\preceq(i,j)}$, $1 \le i, j \le 3$

| Appr. | Rule |
|---|---|
| $\underline{S^{\succeq(3,1)}}$ | if $\Delta_1(x, y) \ge 18.0$ and $\Delta_3(x, y) \ge 94.39$ then $Cl(x) \ge C_3$ and $Cl(y) \le C_1$ |
| $\underline{S^{\succeq(3,2)}}$ | if $\Delta_2(x, y) \ge 10.0$ and $\Delta_3(x, y) \ge 75.35$ then $Cl(x) \ge C_3$ and $Cl(y) \le C_2$ |
| $\underline{S^{\succeq(2,1)}}$ | if $\Delta_1(x, y) \ge 8.67$ and $\Delta_3(x, y) \ge 6.05$ then $Cl(x) \ge C_2$ and $Cl(y) \le C_1$ |
| $\underline{S^{\succeq(3,3)}}$ | if $\Delta_1(x, y) \ge -156.0$ and $\Delta_3(x, y) \ge -148.15$ then $Cl(x) \ge C_3$ and $Cl(y) \le C_3$ |
| $\underline{S^{\succeq(2,2)}}$ | if $\Delta_1(x, y) \ge -12.99$ and $\Delta_3(x, y) \ge -9.0$ then $Cl(x) \ge C_2$ and $Cl(y) \le C_2$ |
| $\underline{S^{\succeq(1,1)}}$ | if $\Delta_1(x, y) \ge -0.43$ then $Cl(x) \ge C_1$ and $Cl(y) \le C_1$ |
| $\underline{S^{\preceq(1,3)}}$ | if $\Delta_1(x, y) \le -18.0$ and $\Delta_3(x, y) \le -94.39$ then $Cl(x) \le C_1$ and $Cl(y) \ge C_3$ |
| $\underline{S^{\preceq(2,3)}}$ | if $\Delta_2(x, y) \le -10.0$ and $\Delta_3(x, y) \le -75.35$ then $Cl(x) \le C_2$ and $Cl(y) \ge C_3$ |
| $\underline{S^{\preceq(1,2)}}$ | if $\Delta_1(x, y) \le -8.67$ and $\Delta_3(x, y) \le -6.05$ then $Cl(x) \le C_1$ and $Cl(y) \ge C_2$ |
| $\underline{S^{\preceq(3,3)}}$ | if $\Delta_1(x, y) \le 156.0$ and $\Delta_3(x, y) \le 148.15$ then $Cl(x) \le C_3$ and $Cl(y) \ge C_3$ |
| $\underline{S^{\preceq(2,2)}}$ | if $\Delta_1(x, y) \le 12.99$ and $\Delta_3(x, y) \le 9.0$ then $Cl(x) \le C_2$ and $Cl(y) \ge C_2$ |
| $\underline{S^{\preceq(1,1)}}$ | if $\Delta_1(x, y) \le 0.43$ then $Cl(x) \le C_1$ and $Cl(y) \ge C_1$ |

**Table 4** Weights of arcs in the preference graph

| $w^{\succeq(3,1)}$ | $w^{\succeq(3,2)}$ | $w^{\succeq(2,1)}$ | $w^{\succeq(1,1)}$ | $w^{\succeq(2,2)}$ | $w^{\succeq(3,3)}$ |
|---|---|---|---|---|---|
| 6 | 3 | 3 | 1 | 1 | 1 |
| $w^{\preceq(1,3)}$ | $w^{\preceq(2,3)}$ | $w^{\preceq(1,2)}$ | $w^{\preceq(1,1)}$ | $w^{\preceq(2,2)}$ | $w^{\preceq(3,3)}$ |
| 6 | 3 | 3 | 1 | 1 | 1 |

- equal importance of preference for pairs of units for which the class difference is the same, e.g., $w^{\succeq(3,2)} = w^{\succeq(2,1)} = 3$.

**Ranking.** In Table 5, we show the result of application of the induced decision rules on set $A$, and aggregation of the resulting relations $\mathbb{S}^{\succeq(i,j)}$ and $\mathbb{S}^{\preceq(i,j)}$ using scoring function $NFS$ with the weights provided in Table 4. The obtained ranking is unique for the induced set of decision rules and adopted weights. For clarity, for each research unit we additionally decompose its comprehensive score into the strength and weakness derived from both $\mathbb{S}^{\succeq(i,j)}$ and $\mathbb{S}^{\preceq(i,j)}$. The final ranking (see Table 5, column $Rank(x)$) reproduces the preference order derived from assignments examples, i.e., all reference units assigned to class $Cl_{h+1}$ are ranked better than these assigned to class $Cl_h$, for $h = 1, 2$. Moreover, when compared with the original classification of the Polish Ministry of Science and Higher Education, all units judged as the best (worst) ones by the Ministry, i.e., $a$ to $c$ ($j$ to $m$), attain clearly positive (negative) scores in our procedure.

**Table 5** Final scores (column $NFS(x)$) and ranks (column $Rank(x)$) of research units (for each unit, we provide its strength and weakness reflected in $NFS(x)$, both with respect to $\mathbb{S}^{\succeq(i,j)}$ and $\mathbb{S}^{\preceq(i,j)}$)

| Unit ($x$) | $\mathbb{S}^{\succeq(i,j)}$ | | $\mathbb{S}^{\preceq(i,j)}$ | | $NFS(x)$ | $Rank(x)$ | $Ref.$ |
|---|---|---|---|---|---|---|---|
| | Strength | Weakness | Strength | Weakness | | | |
| $a$ | 116 | 10 | 117 | 10 | 213 | 1 | $Cl_3$ |
| $b$ | 113 | 9 | 114 | 8 | 210 | 2 | – |
| $c$ | 100 | 8 | 100 | 9 | 183 | 3 | $Cl_3$ |
| $d$ | 41 | 28 | 45 | 30 | 28 | 4 | $Cl_2$ |
| $e$ | 39 | 37 | 41 | 35 | 8 | 5 | – |
| $f$ | 37 | 46 | 38 | 45 | −16 | 7 | $Cl_2$ |
| $g$ | 27 | 42 | 29 | 41 | −27 | 8 | – |
| $h$ | 35 | 31 | 35 | 34 | 5 | 6 | – |
| $i$ | 26 | 54 | 22 | 59 | −63 | 9 | – |
| $j$ | 23 | 72 | 21 | 75 | −103 | 10 | $Cl_1$ |
| $k$ | 16 | 85 | 18 | 84 | −135 | 11 | $Cl_1$ |
| $l$ | 15 | 86 | 16 | 86 | −141 | 12 | – |
| $m$ | 13 | 93 | 13 | 93 | −160 | 13 | – |

## 7 Summary and Conclusions

We presented a new method for multiple criteria ranking problem, characterized by the following features:

- the preference information provided by the DM has the form of sorting examples, i.e., assignments of some reference alternatives to pre-defined and ordered quality classes,
- the intensity of preference between any two alternatives is considered as purely ordinal, i.e., the number of quality classes separating two assigned alternatives is not meaningful for intensity of preference,
- the intensity of preference for pairs of quality classes can be represented by a lattice depicted by Hasse diagram, i.e., one can say that intensity of preference for a pair of alternatives is greater than that of another pair, only if the interval of classes for the first pair includes that of the second pair,
- the method employs the decision rule preference model—the rules are induced from rough approximations of unions of preference intensity relations, without converting the ordinal input preference information into cardinal one,
- the set of rules is an easy to read summary of scenarios of causal relationships between evaluations of pairs of reference alternatives on a subset of criteria and a comprehensive judgment,

- application of decision rules on a considered set of alternatives leads to a preference graph—its exploitation using the weighted net flow score procedure results in a linear ranking.

In conclusion, one can observe that the proposed method does what was promised: starting from an ordinal preference information about intensity of preference on a subset of alternatives, it builds an intelligible preference model being compatible with the input preference information, and applies this model on the whole set of considered alternatives to finally rank them from the best to the worst. An illustrative case study performed at the end of this paper supports this claim.

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